

Electromagnetic Excitation of Rotating Black Holes and Relativistic Jets

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We show that electromagnetic excitations of rotating black holes can lead to the appearance of narrow singular beams which break up the black hole horizon forming a tube-like region which connects the interior and exterior. It is argued that this effect may be at the origin of jet formation.

1. Introduction.

The Kerr solution describes the stationary phases of rotating blackholes. Non-stationary behaviors, like bursts and jets, should of course be described by nonstationary solutions which, however, are not known so far. On the other hand, the Kerr metric is only one representative of the broad Kerr-Schild class of Einstein-Maxwell solutions. Among these solutions there are rotating Kerr-Schild solutions containing axial, semi-infinite singular lines (beams). These solutions have not been paid so far enough attention in astrophysical applications, and thus have never been analyzed in detail from the physical point of view. These solutions however may be considered as a low frequency limit of some wave solutions related to the aligned (coherent) excitations of the rotating black holes. In the paper [1] the structure of horizons for the stationary solutions with axial singularities was considered and some of the astrophysical effects which may be related to the appearance of the axial singularities. It was shown that physical consequences originated from these singularities turn out to be crucial for the initially stable black holes. Even weak axial singularities “break up” the Kerr black hole, forming a “hole in the horizon”. As a result, the internal region turn out to be connected with the external one, and the black hole turns out to be “half dressed”.

In this paper we obtain the exact wave solutions of the Maxwell equations on the Kerr background which are aligned to the Kerr principal null congruence and show that these solutions are the wave generalizations of the corresponding exact stationary Kerr-Schild solutions. In

the quasi-stationary low frequency limit, they tend to the exact self-consistent solutions with an arbitrary degree of approximation in the whole space-time but for the excision of a narrow vicinity of the beam. This leads us to the conclusion that the appearance of the holes in horizon and beams may be caused by a coherent excitation of the rotating sources [1]. The corresponding physical processes may result in the production of jets [2].

2. Solutions with axial singularities.

In the fundamental paper by Debney, Kerr and Schild [3] exact solutions were obtained for the metric form

$$g^{\mu\nu} = \eta^{\mu\nu} - 2Hk^{\mu}k^{\nu}, \quad (1)$$

where k^{μ} is a (1) with a null vector field k^{μ} ($k_{\mu}k^{\mu} = 0$), which is tangent to a geodesic and shear-free principal null congruence (PNC). These spacetimes are algebraically special; as a consequence, many tetrad Ricci components vanish and there is a strong restriction on the tetrad components of the electromagnetic field. The principal property of these solutions is that the electromagnetic field is aligned with the Kerr PNC, satisfying the constraints

$$F^{\mu\nu}k_{\mu} = 0. \quad (2)$$

It is described by two non-vanishing tetrad components of the self-dual tensor $\mathcal{F}_{12} = AZ^2$, $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$, where commas denote the directional derivatives wrt the chosen null tetrad vectors.¹ The resulting equations for the e.m. field are

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}A = 0, \quad \gamma_{,4} = 0, \quad (3)$$

¹The real null tetrad vector $e^3 \equiv e_4 = k_{\mu}dx^{\mu}$.

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0. \quad (4)$$

Gravitational field equations yield

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}M = A\bar{\gamma}\bar{Z}, \quad \mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad (5)$$

where $\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2$.

Solutions of this system were given in [3] only for the stationary case, with $\gamma = 0$. We show that the equations for e.m. field may be integrated for $\gamma \neq 0$. To get an oscillating solution, one defines a complex retarded-time parameter $\tau = t - r + ia \cos \theta$ which satisfies the relations $(\tau)_{,2} = (\tau)_{,4} = 0$. It allows one to represent (3) in the form [4]

$$(AP^2)_{,2} = 0, \quad P_{,2} = -PY_{,3}. \quad (6)$$

This equation can be integrated, yielding $A = \psi(Y, \tau)/P^2$. It has the form obtained in [3]. The only difference is in the extra dependence of the function ψ from the retarded-time parameter τ . It means, that the stationary solutions obtained in [3] may be considered as low-frequency (or adiabatical) limits of these solutions.

One can check that the action of the operator \mathcal{D} on the variables Y, \bar{Y} and $\rho = x^\mu e_\mu^3$ is

$$\mathcal{D}Y = \mathcal{D}\bar{Y} = 0, \quad \mathcal{D}\rho = 1, \quad (7)$$

and therefore $\mathcal{D}\rho = \partial\rho/\partial t_0 \mathcal{D}t_0 = P\mathcal{D}t_0 = 1$, what yields

$$\mathcal{D}t_0 = P^{-1}. \quad (8)$$

As a result, Eqs. (4) take the form

$$\dot{A} = -(\gamma P)_{,\bar{Y}}, \quad \gamma_{,4} = 0, \quad (9)$$

where $(\dot{}) \equiv \partial_{t_0}$.

For the stationary background considered here, $P = 2^{-1/2}(1 + Y\bar{Y})$, and $\dot{P} = 0$. The coordinates Y , and τ are independent from \bar{Y} , which allows us to integrate (9). We obtain the following general solution

$$\gamma = -P^{-1} \int \dot{A} d\bar{Y} = \frac{2^{1/2}\dot{\psi}}{P^2 Y} + \phi(Y, \tau)/P, \quad (10)$$

where ϕ is an arbitrary analytic function of Y and τ . The term γ in $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$ describes a part of the null electromagnetic radiation which falls off asymptotically as $1/r$ and propagates along the Kerr principal null congruence e^3 . It follows from (5) that γ describes a loss of mass by radiation with the stress-energy tensor $\kappa T_{\mu\nu}^{(\gamma)} = \frac{1}{2}\gamma\bar{\gamma}e_\mu^3 e_\nu^3$.

We now evaluate the term $(AZ)_{,1}$. For the stationary case we have the relations $Z_{,1} = 2ia\bar{Y}(Z/P)^3$ and $\tau_{,1} = -2ia\bar{Y}Z/P^2$. This yields

$$(AZ)_{,1} = \frac{Z}{P^2}(\psi_{,Y} - 2ia\dot{\psi}\frac{\bar{Y}}{P^2} - 2\psi\frac{P_Y}{P}) + A2ia\frac{Z\bar{Y}}{P^3}. \quad (11)$$

Since $Z/P = 1/(r + ia \cos \theta)$, this expression contains terms which fall off like r^{-2} and r^{-3} . However, it contains also factors which depend on the coordinate $Y = e^{i\phi} \tan \frac{\theta}{2}$ and can be singular at the z -axis, forming narrow beams, i.e. the half-infinite lines of singularity. In particular, these lines can be the z^+ or z^- axis, which correspond to $\theta = 0$ and $\theta = \pi$ (cases $n = \pm 1$, respectively).

The exact Debney-Kerr-Schild solutions arise in the limit $\gamma = 0$, which yields a constant electromagnetic field. The unique non-zero component of the field tensor in this case is $\mathcal{F}_{31} = -(AZ)_{,1}$ where Z is the (complex) expansion of the PNC. The function A has the general form

$$A = \psi(Y)/P^2, \quad (12)$$

where $P = 2^{-1/2}(1 + Y\bar{Y})$, and ψ is an arbitrary holomorphic function of Y . The resulting metric has the Kerr-Schild form (1), where the function h is given by [3]

$$h = m(Z + \bar{Z})/(2P^3) - A\bar{A}Z\bar{Z}/2. \quad (13)$$

In terms of spherical coordinates on the flat background one has $Y(x) = e^{i\phi} \tan \frac{\theta}{2}$, which is singular at $\theta = \pi$. This singularity will be present in any holomorphic function $\psi(Y)$, and, consequently, in A and in h . Therefore, all the solutions of this class—with the exclusion of the case $\psi = \text{const}$ which corresponds to the Kerr-Newman solution—will be singular at some angular direction θ .

The simplest cases are $\psi = q/(Y + c)$ and $\psi = q(Y + b)/(Y + c)$, which correspond to an arbitrary direction of the axial singularity. However, the sum of singularities in different directions is also admissible $\psi(y) = \sum_i q_i(Y + b_i)/(Y + c_i)$, as well as polynomials of higher degree. Notice, that the axial singularity survives for arbitrarily small values of q when $q \neq 0$. It yields the solutions which tend to the exact one in the limit $q/m \rightarrow 0$ everywhere but for the exclusion of an ϵ -vicinity of the axial singularity.

In the quasi-stationary limit, $\dot{A} \rightarrow 0$, from Eq.(9) it follows that the solutions correspond to the well known $\gamma = 0$ solutions.

3. Causal structure.

The properties of the horizons of these solutions were considered in [1] and it was shown that the black holes turn out to be broken up by the axial singularities, with the appearance of a tubelike region which connects internal and external regions, allowing matter to escape. As a result, the circular Kerr singularity turns out to be “half-dressed”. These solutions do not contradict the assertion of the “No hair theorem” of Carter and Robinson, which states uniqueness of the Kerr and Kerr-Newman solutions [5,6] under certain *regularity* hypotheses, since these hypothesis are here *not* satisfied [1]. The structure of the horizons for the solutions containing axial singular lines follows from the form of metric (1) where k^μ is a

null vector field $k_\mu k^\mu = 0$ which is tangent to the Kerr principal null congruence. The function H has the form

$$H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \quad (14)$$

where the oblate coordinates r, θ are used on the flat Minkowski background $\eta^{\mu\nu}$. In the case of rotating Kerr solutions, the Schwarzschild horizon splits into four surfaces. Two surfaces correspond to the staticity limit, r_{s+} and r_{s-} , which are determined by the condition $g_{00} = 0$, and two surfaces come from the event and Cauchy horizons, $S(x^\mu) = \text{const.}$, which are the null surfaces determined by the condition $g^{\mu\nu}(\partial_\mu S)(\partial_\nu S) = 0$. In the case $e^2 + a^2 > M^2$, the horizons of the Kerr-Newman solution disappear and the Kerr singular ring turns out to be naked. The simplest axial singularity is the pole $\psi = q/Y$. In this case the boundaries of the ergosphere, r_{s+} and r_{s-} , are determined by the condition $g_{00} = 0$ and the solution acquires a new feature: the surfaces r_{s+} and r_{s-} turn out to be joined by a tube, forming a simply connected surface.

The surfaces of the *event horizons* are null and obey the differential equation

$$(\partial_r S)^2 [r^2 + a^2 + (q/\tan \frac{\theta}{2})^2 - 2Mr] - (\partial_\theta S)^2 = 0. \quad (15)$$

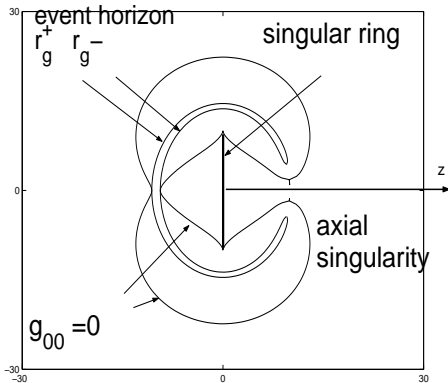


FIG. 1. Near extremal black hole with a hole in the horizon, for $m = 10$, $a = 9.98$, $q = 0.1$. The event horizon is a closed connected surface surrounded by the closed connected surface $g_{00} = 0$.

Similar to the boundary of the ergosphere, the two event horizons are joined into one connected surface, and the surface of the event horizon lies inside the boundary of the ergosphere. The resulting surfaces are shown in Figs. 1, while other examples can be found in [1]. As it is seen from the figure, the axial singularities lead to the formation of the holes in the black hole horizon, which opens the interior of the “black hole” up to external space.

The structure of the diagrams of the maximal analytic extension (MAE) was also discussed in [1]. It depends on the section considered. If the section is chosen to be away from the corresponding tube-like region, the diagram of

the MAE will be just the same as for the usual solution for a rotating black hole. If the section goes through the axial singularity, the tube-like hole in the horizon leaves a trace on all patches of the MAE. The r_+ and r_- surfaces are deformed and approach towards each other, joining at some distance from the axial singularity and forming the tube-like channels connecting the interior and the exterior at some angular direction will appear on all patches of the diagram.

These *black holes with holes in the horizon* have thus *preferred directions* along which the causal structure differs from that of “true black holes”. Their singularity is, therefore, naked, but the nakedness is of a very peculiar type, since it manifests itself in specific directions only. A similar situation occurs with other non-spherical exact solutions, like e.g. the so called Gamma metric [7].

4. Possible astrophysical consequences.

Axial singularities carry travelling electromagnetic and gravitational waves which propagate along them as along a waveguide, a phenomenon described by exact singular *pp*-wave solutions of the Einstein-Maxwell field equations [8]. The appearance of the axial singularities in rotating astrophysical sources may be related to their *excitations* by gravitational and/or electromagnetic waves, and has to be necessarily caused by some non-stationary process. It was argued in [1,8] that e.m. excitation of black holes leads inevitably to the appearance of axial singularities.

The simplest wave modes

$$\psi_n = qY^n \exp i\omega_n \tau \equiv q(\tan \frac{\theta}{2})^n \exp i(n\phi + \omega_n \tau) \quad (16)$$

can be labelled by the index $n = \pm 1, \pm 2, \dots$, which corresponds to the winding number for the phase wrapped around the axial singularity. The leading wave terms have the form $\mathcal{F}|_{\text{wave}} = f_R d\zeta \wedge du + f_L d\zeta \wedge dv$, where $f_R = (AZ)_{,1}$ and $f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1}$ are the factors describing the “left” and “right” waves propagating, correspondingly, along the z^- and z^+ semi-axes. Near the z^+ axis, $|Y| \rightarrow 0$, and for $r \rightarrow \infty$, we have $Y \simeq e^{i\phi} \frac{\rho}{2r}$, where ρ is the distance from the z^+ axis. Similarly, near the z^- axis $Y \simeq e^{i\phi} \frac{2r}{\rho}$ and $|Y| \rightarrow \infty$. For $|n| > 1$ the solutions contain axial singularities which do not fall off asymptotically, but are increasing, denoting instability. For example, the leading wave for $n = -1$,

$$\mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi + i\omega_{-1}\tau}}{\rho^2} d\zeta \wedge du, \quad (17)$$

is singular at the z^+ semi-axis and propagates to $z = +\infty$. The wave excitations of the Kerr geometry may lead to the appearance of two singular *pp*-waves which propagate outward along the axial singularities. In real situations, axial singularities cannot be stable and they will presumably correspond to some type of jet or burst, and hence it is natural to conjecture that the related holes in the horizon may be at the origin of jet formation.

Observational evidence shows a preference for two-jet-like sources, as e.g., in the field of radio loud sources [9,10]. These jets are emitted in opposite directions along the same axis. This scenario corresponds to the sum of two singular modes with $n = \pm 1$ and to two opposite positioned holes in the horizon, see Fig.3.

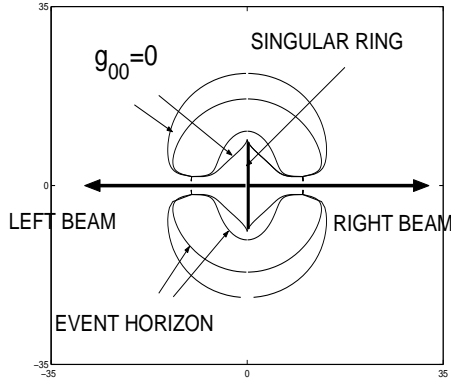


FIG. 2. Bi-directional singular beams forming two holes in the horizon. Parameters: $m = 10$, $a = 9$, $q = 0.3$.

Electromagnetic pp -waves along the singularity will cause a strong longitudinal pressure pointed outwards from the hole. It can be easily estimated for the modes of the pp -waves with $n = \pm 1$ [8]. For example, the corresponding energy-momentum tensor is $T^{\mu\nu} = \frac{1}{8\pi} |\mathcal{F}_{-1}^+|^2 k^\mu k^\nu$, and the wave beam with mode $n = -1$, propagating along the z^+ half-axis, will exert the pressure

$$p_{z^+} = \frac{2q^2 e^{2a\omega_{-1}}}{\pi \rho^4}, \quad (18)$$

where ρ is an axial distance from the singularity and ω_{-1} is the frequency of this mode. For the exact stationary Kerr-Schild solutions, one can use this expression in the limit $\omega_{-1} = 0$.

5. Conclusions.

From the analysis above, we conclude that the aligned excitations of the rotating black hole (or naked rotating source) lead, unavoidably, to the appearance of axial singularities accompanied by outgoing travelling waves and also to the formation of holes at the horizon, which on its turn can lead to the production of astrophysical jets [2].

Multiparticle Kerr-Schild solutions [11] suggest that axial singularities will be bi-directional and oriented along the line connecting the interacting particles. Thus, it will be interesting to analyze in further detail the observed jets in order to check the conjecture that they may be indeed triggered by radiation coming from remote active objects.

As far as the axial singularity survives for arbitrarily small $q/m \neq 0$, one can consider also the case of small quantum excitations. One expects that by elementary

quantum excitations the axial singularities shall lead to the formation of very small quantum holes with a (maybe just momentary) opening of the horizon. This effect may be related to the origin of Hawking's quantum evaporation.

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